

Shielding from Space Radiations

A Final Progress Report

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## ABSTRACT

This Final Progress Report for NCC-1-178 presents the details of the engineering developement of an analytical/computational solution to the heavy ion transport equation in terms of a multi-layer Green's function formalism as applied to the Small Spacecraft Technology Initiative (SSTI) program.

The mathematical developments are recasted into a series of efficient computer codes for space applications. The efficency of applied algorithms is accomplished by a nonperturbative technique of extending the Green's function over the solution domain. The codes may also be applied to the accelerator boundary conditions to allow code validation in laboratory experiments.

Correltions with experiments for the isotopic version of the code with 59 and 80 isotopes present for a two layers target material in water has been verified.

## INTRODUCTION

Future NASA missions will be limited by exposure to space radiations unless adequate shielding is provided to protect men and equipments from such radiations. Adequate methods required to estimate the damage caused by such radiations behind various shields can be evaluated prior to commitment to such missions.

From the inception of the Langley Research Center heavy ion (HZE) shielding program (refs. 2-4), there has been a continued, close relationship between code development and laboratory experiment (ref. 4). Indeed, the current research goal is to provide computationally efficient high charge and energy ion (HZE) transport codes which can be validated with laboratory experiments and subsequently applied to space engineering design. In practice, two streams of code development have prevailed due to the strong energy dependence of necessary atomic/molecular cross sections and the near singular nature of the laboratory beam boundary conditions (refs. 5-7). The atomic/molecular cross section dependence is adequately dealt with by using the methods of Wilson and Lamkin (ref. 8), allowing efficient numerical procedures to be developed for space radiations (refs. 7,9-11). Although these codes could conceivably be applied to the laboratory validation, methods to control truncation and discretization errors would bear little resemblance to the space radiation codes attempting to be validated. Clearly, a radical reorientation is required to achieve the validation goals of the current NASA space radiation shielding program, and such an approach in terms of a multi-layered Green's function formalism as an extension to a previously developed single-layered algorithm is the main thrust of this continuing research and is briefly described below.

## GREEN'S FUNCTION METHODS

### Transport Equations and Conventional Approach

The transport equation for high energy heavy ions is usually simplified by assuming the straightahead approximation and neglecting the target secondary fragments (ref. 3 et. al) and is written as

$$\left[ \frac{\partial}{\partial x} - \frac{\partial}{\partial E} \tilde{S}_j(E) + \sigma_j \right] \phi_j(x, E) = \sum_k \sigma_{jk} \phi_k(x, E) \quad (1)$$

where  $\phi_j(x, E)$  is the ion flux at  $x$  with energy  $E$  (MeV/amu),  $\tilde{S}_j(E)$  is the change in  $E$  per unit distance,  $\sigma_j$  the total macroscopic absorption cross section and  $\sigma_{jk}$  the macroscopic cross section for collision of ion type  $k$  to produce an ion of type  $j$ . The solution to equation (1) is to be found subject to the boundary condition

$$\phi_j(0, E) = f_j(E) \quad (2)$$

which for laboratory beams has only one value of  $j$  for which  $f_j(E)$  is not zero and that  $f_j(E)$  is described by a mean energy spread  $\sigma$  such that

$$f_j(E) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ -(E - E_o)^2 / 2 \sigma^2 \right] \quad (3)$$

The solution to equation (1) is given by superposition of Greens function  $G_{jk}$  as

$$\phi_j(x, E) = \sum_k \int G_{jk}(x, E, E') f_k(E') dE' \quad (4)$$

where Green function is a solution of

$$\left[ \frac{\partial}{\partial x} - \frac{\partial}{\partial E} \tilde{S}_j(E) + \sigma_j \right] G_{jm}(x, E, E_o) = \sum_k \sigma_{jk} G_{km}(x, E, E_o) \quad (5)$$

subject to the boundary condition

$$G_{jm}(0, E, E_0) = \delta_{jm} \delta(E - E_0) \quad (6)$$

The above equations can be simplified by transforming the energy into the residual range as

$$r_j = \int_0^E dE' / \tilde{S}_j(E') \quad (7)$$

and defining new field functions as

$$\psi_j(x, r_j) = \tilde{S}_j(E) \phi_j(x, E) \quad (8)$$

$$G_{jm}(x, r_j, r'_m) = \tilde{S}_j(E) G_{jm}(x, E, E') \quad (9)$$

$$\hat{f}_j(r_j) = \tilde{S}_j(E) f_j(E) \quad (10)$$

equation (5) becomes

$$\left[ \frac{\partial}{\partial x} - \frac{\partial}{\partial r_j} + \sigma_j \right] G_{jm}(x, r_j, r'_m) = \sum_k \frac{\nu_j}{\nu_k} \sigma_{jk} G_{km}(x, r_k, r'_m) \quad (11)$$

with the boundary condition

$$G'_{jm}(0, r_j, r'_m) = \delta_{jm} \delta(r_j - r'_m) \quad (12)$$

and with the solution to the ion fields given by

$$\psi_j(x, r_j) = \sum_m \int_0^\infty G_{jm}(x, r_j, r'_m) \hat{f}_m(r'_m) dr'_m \quad (13)$$

Note  $\nu_j$  is the range scale factor as  $\nu_j r_j = \nu_m r_m$  and is taken as  $\nu_j = Z_j^2 / A_j$ . The solution to equation (11) is written as a perturbation series

$$G_{jm}(x, r_j, r'_m) = \sum_i G_{jm}^{(i)}(x, r_j, r'_m) \quad (14)$$

where

$$G_{jm}^{(0)}(x, r_j, r'_m) = g(j) \delta_{jm} \delta(x + r_j - r'_m) \quad (15)$$

and

$$G_{jm}^{(1)}(x, r_j, r'_m) \approx \frac{\nu_j \sigma_{jm} g(j, m)}{x(\nu_m - \nu_j)} \quad (16)$$

where  $G_{jm}^{(1)}(x, r_j, r'_m)$  is zero unless

$$\frac{\nu_j}{\nu_m} (r_j + x) \leq r'_m \leq \frac{\nu_j}{\nu_m} r_j + x \quad (17)$$

for  $\nu_m > \nu_j$ . If  $\nu_j > \nu_m$  as can happen in neutron removal, the negative of equation (16) is used and upper and lower limits of equation (17) are switched. The higher order terms are approximated as

$$G_{jm}^{(i)}(x, r_j, r'_m) \approx \sum_{k_1, k_2, \dots, k_{i-1}} \frac{\nu_j \sigma_{jk_1} \sigma_{k_1 k_2} \dots \sigma_{k_{i-1} m} g(j, k_1, k_2, \dots, k_{i-1}, m)}{x(\nu_m - \nu_j)} \quad (18)$$

In the above

$$g(j) = e^{-\sigma_j x} \quad (19)$$

and

and

$$g(j_1, j_2, \dots, j_n, j_{n+1}) = \frac{g(j_1, j_2, \dots, j_{n-1}, j_n) - g(j_1, j_2, \dots, j_{n-1}, j_{n+1})}{\sigma_{j_{n+1}} - \sigma_{j_n}} \quad (20)$$

Note that  $\mathcal{G}_{jm}^{(i)}(x, r_j, r'_m)$  is purely dependent on  $x$  for  $i > 0$ , which we represent as  $\mathcal{G}_{jm}^{(i)}(x)$ . (See ref. 3.) In terms of the above, the solution to equation (1) becomes (from ref. 3)

$$\begin{aligned} \psi_j(x, r_j) = & e^{-\sigma_j x} \hat{f}_j(r_j + x) \\ & + \sum_{m,i} \mathcal{G}_{jm}^{(i)}(x) \left[ \hat{F}_m(r'_{m\ell}) - \hat{F}_m(r'_{mu}) \right] \end{aligned} \quad (21)$$

In equation (21),  $r'_{mu}$  and  $r'_{m\ell}$  are given by the upper and lower limits of the inequality of equation (17). The symbol  $\hat{F}_m(r'_m)$  refers to the integral spectrum

$$\hat{F}_m(r'_m) = \int_{r'_m}^{\infty} \hat{f}_m(r) dr$$

We note that

$$\hat{F}_m(r'_m) \equiv F_m(E')$$

with

$$F_m(E') = \int_{E'}^{\infty} f_m(E) dE \quad (22)$$

and

$$r'_m = \int_0^{E'} dE / \tilde{S}_m(E)$$

We now introduce nonperturbative terms for the summation in equation (21).

First, we recall that the  $g$  function of  $n$  arguments was generated by the perturbation solution of the transport equation neglecting ionization energy loss (ref. 1) given by

$$\left[ \frac{\partial}{\partial x} + \sigma_j \right] g_{jm}(x) = \sum_k \sigma_{jk} g_{km}(x) \quad (23)$$

subject to the boundary condition

$$g_{jm}(0) = \delta_{jm} \quad (24)$$

for which the solution is

$$g_{jm}(x) = \delta_{jm} g(m) + \sigma_{jm} g(j, m) + \dots \quad (25)$$

It is also true that

$$g_{jm}(x) = \sum_k g_{jk}(x - y) g_{km}(y) \quad (26)$$

for any positive values of  $x$  and  $y$ . Equation (26) may be used to propagate the function  $g_{jm}(x)$  over the solution space from very thin shield solutions. Equation (14) is then rewritten as

$$g_{jm}(x, r_j, r'_m) \approx e^{-\sigma_j x} \delta_{jm} \delta(x + r_j - r'_m) + \nu_j [g_{jm}(x) - e^{-\sigma_j x} \delta_{jm}] / [x(\nu_m - \nu_j)] \quad (27)$$

and the approximate solution of equation (1) is given by

$$\begin{aligned} \psi_j(x, r_j) = & e^{-\sigma_j x} \hat{f}(r_j + x) \\ & + \sum_m \frac{\nu_j [g_{jm}(x) - e^{-\sigma_j x} \delta_{jm}]}{x(\nu_m - \nu_j)} [\hat{F}_m(r'_{mu}) - \hat{F}_m(r'_{ml})] \end{aligned} \quad (28)$$

### Green's Function Methods in a Shielded Medium

The major simplification in the Green's function method results from

the fact that the scaled spectral distribution of secondary ions to a first approximation depends only on the depth of target penetration as seen in equations (16), (18), and (27). The first approach to a multi-layered Green's function will rely on this observation and assume its validity for multi-layered shields.

Consider a domain labeled as 1 which is shielded by a second domain labeled as 2, the number of type  $j$  ions at depth  $x$  in 1 due to type  $m$  ions incident on domain 2 of thickness  $y$  is

$$g_{12jm}(x, y) = \sum_k g_{1jk}(x) g_{2km}(y) \quad (29)$$

The leading term in equation (29) is the penetrating primaries as

$$g_{12jm}(x, y) = e^{-\sigma_{1j}x - \sigma_{2j}y} \delta_{jm} + [g_{12jm}(x, y) - e^{-\sigma_{1j}x - \sigma_{2j}y} \delta_{jm}] \quad (30)$$

where all higher order terms are in the bracket of equation (30).

The first term of scaled Green's function is then

$$G_{12jm}^{(o)}(x, y, r_j, r'_m) = e^{-\sigma_{1j}x - \sigma_{2j}y} \delta_{jm} \delta[x + r_j - (r'_m - \rho y)] \quad (31)$$

where  $\rho$  is the range scale factor for the two media

$$\rho = R_{1j}(E)/R_{2j}(E) \quad (32)$$

A single value for  $\rho$  is taken corresponding to 600 MeV/amu. The secondary contribution is similarly found by noting the equation (17) becomes

$$\frac{\nu_j}{\nu_m}(r_j + x + \rho y) \leq r'_m \leq \frac{\nu_j}{\nu_m} r_j + x + \rho y \quad (33)$$

from which the average spectrum is evaluated. The full approximate Green's function is then

$$G_{12jm}(x, y, r_j, r'_m) \approx e^{-\sigma_{1j}x - \sigma_{2j}y} \delta_{jm} \delta(x + \rho y + r_j - r'_m) + \nu_j \left[ g_{12jm}(x, y) - e^{-\sigma_{1j}x - \sigma_{2j}y} \delta_{jm} \right] / [(x + \rho y)(\nu_m - \nu_j)] \quad (34)$$

Equation (34) is the first approximation to the Green's function in a shielded medium (two layers) and can be modified to multiple layers. The first spectral modification is considered next. It is easy to show that the first collision term has the properties

$$G_{12jm}^{(1)}(x, y, r_j, r'_m) = \begin{cases} \frac{\nu_j \sigma_{1jm} e^{-\sigma_{1m}x - \sigma_{2m}y}}{|\nu_m - \nu_j|} & \text{for } r'_m = r'_{mu} \\ \frac{\nu_j \sigma_{2jm} e^{-\sigma_{1j}x - \sigma_{2j}y}}{|\nu_m - \nu_j|} & \text{for } r'_m = r'_{m\ell} \end{cases} \quad (35)$$

These properties are used to correct the average spectrum as

$$G_{12jm}^{(1)}(x, y, r_j, r'_m) = \frac{\nu_j g_{12jm}^{(1)}(x, y)}{|\nu_m - \nu_j|(x + \rho y)} + b_{jm}(x, y)(r'_m - \bar{r}_m) \quad (36)$$

where  $g_{12jm}^{(1)}(x, y)$  is the first collision term of equation (34) and

$$\bar{r}_m = (r'_{mu} + r'_{m\ell})/2 \quad (37)$$

is the midpoint of  $\bar{r}_m$  between its limits given by equation (33). The term of equation (36) has the property that

$$\int_{r'_{m\ell}}^{r'_{mu}} b_{jm}(x, y)(r' - \bar{r}_m) dr' = 0 \quad (38)$$

ensuring that the first term of equation (36) is indeed the average spectrum as required. The spectral slope parameter is found to be

$$b_{jm}(x, y) = \nu_j \nu_m (\sigma_{1jm} e^{-\sigma_{1m} x - \sigma_{2m} y} - \sigma_{2jm} e^{-\sigma_{1j} x - \sigma_{2j} y}) / [(x + \rho y)(\nu_m - \nu_j)|\nu_m - \nu_j|] \quad (39)$$

A similarly simple spectral correction could be made to the higher order terms. The spectral correction given in equation (39) is included in the present Green's function code.

### Solution for Laboratory Beams

Using the boundary condition appropriate for laboratory beams given by equation (3), cumulative spectrum given by

$$F_j(E) = \frac{1}{2} \left[ 1 - \operatorname{erf} \left( \frac{E - E_o}{\sqrt{2} \sigma} \right) \right] \quad (40)$$

and the cumulative energy moment needed to evaluate the spectral correction given by

$$\bar{E}_j(E) = \frac{1}{2} E_o \left[ 1 - \operatorname{erf} \left( \frac{E - E_o}{\sqrt{2} \sigma} \right) \right] + \frac{\sigma}{\sqrt{2\pi}} \exp \left[ -\frac{(E - E_o)^2}{2\sigma^2} \right] \quad (41)$$

The average energy on any subinterval  $(E_1, E_2)$  is then

$$\bar{E} = [\bar{E}_j(E_1) - \bar{E}_j(E_2)] / [F_j(E_1) - F_j(E_2)] \quad (42)$$

and the beam generated flux becomes

$$\begin{aligned}
\psi_j(x, y, r_j) &= e^{-\sigma_{1j}x - \sigma_{2j}y} \hat{f}_j(r_j + x + \rho y) \\
&+ \sum_{m,i} g_{jm}^{(i)}(x, y) \left[ \hat{F}_m(r'_{mu}) - \hat{F}_m(r'_{m\ell}) \right] \\
&+ \sum_m b_{jm}^{(1)}(x, y) \left[ r'_m(\bar{E}) - \bar{r}'_m \right] \left[ \hat{F}_m(r'_{mu}) - \hat{F}_m(r'_{m\ell}) \right]
\end{aligned} \tag{43}$$

where  $\bar{E}$  is evaluated using equation (42) with  $E_1$ , and  $E_2$  as the lower and upper limit associated with  $r'_{m\ell}$  and  $r'_{mu}$ .

## DISCUSSION OF RESULTS

A series of algorithms in support of Small Spacecraft Technology Initiative (SSTI) based on the multi-layered Green's function formalism were developed and tested during this research project.

In order to implement the theoretical formulations presented in the theory section of this research grant for multi-layered targets, the following tasks were undertaken:

1. A direct conversion from formulation to coding based on the described procedures was performed. This resulted in a first generation multi-layered computer codes with acceptable computational efficiency for the low order terms.
2. Within the scope of the grant, Further improvements were carried out to make the higher order terms, as computationally efficient as possible. The current version of these codes are computationally efficient for nearly all terms.
3. The resulting coded package were further tested against the results of a recent two-layered experiments for correlation purposes and code efficiency studies.
4. The tested version of the two-layered algorithm was used as a mean of evaluating the shielding properties of different two-layered target materials as they are exposed to galactic cosmic radiations (GCR).

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